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Large Numbers Hypothesis. II. Electromagnetic Radiation

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This paper develops the theory of electromagnetic radiation in the units covariant formalism incorporating Dirac's large numbers hypothesis (LNH). A direct field-to-particle technique is used to obtain the photon propagation equation which explicitly involves the photon replication rate. This replication rate is fixed uniquely by requiring that the form of a free-photon distribution function be preserved, as required by the 2.7 K cosmic radiation. One finds that with this particular photon replication rate the units covariant formalism developed in Paper I actually predicts that the ratio of photon number to proton number in the Universe varies as $t^{1/4}$ precisely in accord with LNH. The cosmological red-shift law is also derived and it is shown to differ considerably from the standard form of $\nu R = \text{const}$.

1. INTRODUCTION

This paper is the second in a series seeking to explore the consequences for physics of developing a viable, self-consistent, physical theory incorporating Dirac's (1937) large numbers hypothesis (LNH). In Paper I (Adams 1982) LNH was presented, the guiding principle of units covariance was developed, and a scalar "field" $\varphi(x)$ which possessed certain unusual properties was introduced. In this paper I develop the theory of electromagnetic radiation in the units covariant formalism.

Because of recent (Steigman, 1978; Canuto and Hsieh, 1978, 1979) conflicting reports in the literature concerning the red-shift law, the blackbody spectrum, photon creation, and scale covariance, I present a detailed account of a field-to-particle technique (Robertson and Noonan 1968)

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which allows one to sensibly and self-consistently incorporate photon replication into the units covariant formalism. The photon replication rate is *a priori* unknown. However, if one assumes that the existence of the 2.7 K cosmic radiation requires the preservation of the form of the free-photon distribution function then the photon replication rate is determined uniquely with no input from LNH. The theory then *predicts* that the ratio of photon number to proton number in the Universe varies as $t^{1/4}$ in the multiplicative creation version of LNH, exactly as LNH requires (Adams 1982). This remarkable result is achieved without prior use of this *a priori* independent large number.

The field-to-particle technique is applied to matter and is used to derive the in-geodesic equation as a consistency check. The technique is then used to derive the photon propagation equation which depends on the photon replication rate. Finally, the photon propagation equation is used to derive the cosmological red-shift law which differs from versions previously appearing in the literature (Canuto and Hsieh, 1978, 1979; Dirac, 1974; Canuto *et al.*, 1977). This result is of vital importance for future cosmological applications.

The reader unfamiliar with sign conventions, notation, or the units covariant formalism is referred to Paper I for details. The powers of various quantities are collected here for convenience:

$$\Pi(g_{\mu\alpha}) = +2, \qquad \Pi(g^{\mu\alpha}) = -2, \qquad \Pi(u^\alpha) = -1 \tag{1a}$$

$$\Pi(\varphi) = \Pi(dx^\alpha) = 0, \qquad \Pi(\beta) = -1 \tag{1b}$$

$$\Pi(G) = g, \qquad \Pi(L) = +1, \qquad \Pi(p^\alpha) = -g \tag{1c}$$

$$\Pi(M) = 1 - g, \qquad \Pi(T^{\mu\alpha}) = -4 - g \tag{1d}$$

2. PROPAGATION EQUATIONS

2.1. Matter Propagation Equation. One can obtain the particle propagation equation from the known properties of the energy tensor $T^{\mu\alpha}$ for the appropriate field (Robertson and Noonan, 1968). This field-to-particle technique will be presented here in detail and applied to matter particles to obtain the in-geodesic equation for matter as an example to illustrate its effectiveness. It will then be applied to photons to obtain the photon propagation equation.

Let a curve C be constructed so that it always lies inside the space-time region where $T^{\mu\alpha} \neq 0$, i.e., $T^{\mu\alpha} \neq 0$ on C. Along C one can always construct

Gaussian coordinates so that the line element takes the form

$$d\tau^2 = dt^2 - h_{ij}dx^i dx^j, \qquad (-g)^{1/2} = h^{1/2} \tag{2}$$

The four-dimensional volume element has the form

$$d_4V \equiv \frac{1}{4!}(-g)^{1/2}\varepsilon_{\mu\alpha\sigma\lambda}dx^\mu dx^\alpha dx^\sigma dx^\lambda \tag{3}$$

where $\varepsilon_{\mu\alpha\sigma\lambda}$ is the alternating symbol with $\varepsilon_{0123} = +1$. On C one has

$$d_4V = \frac{1}{3!}h^{1/2}\varepsilon_{ijk}dx^i dx^j dx^k dt \equiv dV\,dt \tag{4}$$

with $\varepsilon_{123} = 1$, and where dV is the element of three-dimensional proper volume of any $t = \text{const}$ hypersurface on C. Let C be given parametrically by coordinates $x^\mu = \xi^\mu(\kappa)$, where κ is any strictly monotone parameter. Then the tangent vector to C is given by

$$r^\mu \equiv \frac{d\xi^\mu}{d\kappa} \tag{5}$$

Let C be surrounded by an arbitrarily small world-tube. At any given time t the three-dimensional volume of this world tube is V. Assume that $T^{\mu\alpha}$ is zero on the boundary of the world tube (the matter is concentrated along C). Assume further that at any time t one can choose V so small that for any arbitrary function $\omega(x)$ possessing a Taylor's series expansion about C one has

$$r^0\int_V \omega(x)T^{\mu\alpha}(x)\,dV = \omega(t, \xi^i)t^{\mu\alpha}(t) \tag{6}$$

Notice that since d_4V of (4) is a scalar and $d\kappa$ is a scalar then $d_4V/d\kappa = r^0\,dV$ is a scalar. Hence the tensorial properties of $t^{\mu\alpha}$ are identical to the tensorial properties of $T^{\mu\alpha}$ on C.

The energy tensor for any closed system satisfies $T^{\mu\alpha}{}_{*\alpha} = 0$. From (1d) one finds

$$\frac{1}{h^{1/2}}(h^{1/2}T^{\mu\alpha})_{,\alpha} = -T^{\lambda\alpha}\Gamma^\mu_{\lambda\alpha} + (g-2)T^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta} + T^\lambda{}_\lambda g^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta} \tag{7}$$

where (2) was used. Multiply (7) by the arbitrary function $\omega(x)$ to get

$$\frac{1}{h^{1/2}}(h^{1/2}\omega T^{\mu\alpha})_{,\alpha} = \omega_{,\alpha}T^{\mu\alpha} + \omega\left[-T^{\lambda\alpha}\Gamma^\mu_{\lambda\alpha} + (g-2)T^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta} + T^\lambda{}_\lambda g^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta}\right] \tag{8}$$

Applying (6) to (8) gives

$$r^0\int_V \frac{1}{h^{1/2}}\frac{\partial}{\partial x^\alpha}(h^{1/2}\omega T^{\mu\alpha})\,dV = \omega_{,\alpha}t^{\mu\alpha} + \omega\left[-t^{\lambda\alpha}\Gamma^\mu_{\lambda\alpha} + (g-2)t^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta} + t^\lambda{}_\lambda g^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta}\right] \tag{9}$$

so use of (4) gives

$$\begin{aligned} r^0\int_V \frac{1}{h^{1/2}}\frac{\partial}{\partial x^\alpha}(h^{1/2}\omega T^{\mu\alpha})\,dV &= r^0\frac{d}{dt}\int_V \omega T^{\mu 0}\,dV \\ &\quad + r^0\int_V \frac{\partial}{\partial x^n}(h^{1/2}\omega T^{\mu n})\varepsilon_{ijk}\,dx^i\,dx^j\,dx^k \\ &= \frac{d}{d\kappa}\left(\frac{\omega t^{\mu 0}}{r^0}\right) = \frac{t^{\mu 0}}{r^0}\omega_{,\alpha}r^\alpha + \omega\frac{d}{d\kappa}\left(\frac{t^{\mu 0}}{r^0}\right) \end{aligned} \tag{10}$$

where the second integral of (10) vanishes because $T^{\mu\alpha}$ is zero on the boundary of V. Combining (10) with (9) and using the arbitrariness of $\omega(x)$ gives

$$\frac{d}{d\kappa}\left(\frac{t^{\mu 0}}{r^0}\right) + t^{\lambda\alpha}\Gamma^\mu_{\lambda\alpha} + (2-g)t^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta} - t^\lambda{}_\lambda g^{\mu\alpha}\frac{\beta_{,\alpha}}{\beta} = 0 \tag{11a}$$

$$\frac{t^{\mu 0}}{r^0}r^\alpha = t^{\mu\alpha} \tag{11b}$$

Since r^α and $t^{\mu\alpha}$ are tensors on C,

$$t^\mu \equiv t^{\mu 0}/r^0 \tag{12}$$

must be a vector on C from (11b). Use of (12), (11b), (5), (3), and (1) gives

$$\begin{aligned} \Pi(t^\mu) &= \Pi(t^{\mu\alpha}) - \Pi(r^\alpha) = \Pi(T^{\mu\alpha}) + \Pi(r^0 dV) - \Pi(r^\alpha) \\ &= -4 - g + \Pi(d_4V) - \Pi(d\kappa) - \Pi(d\xi^\alpha) + \Pi(d\kappa) = -g \end{aligned} \tag{12a}$$

so (11) becomes

$$t^{\mu}{}_{*\alpha} r^{\alpha} = (t^{\alpha} r^{\mu} - t^{\mu} r^{\alpha}) \beta_{,\alpha} / \beta = 0 \tag{13a}$$

$$t^{\mu} r^{\alpha} = t^{\mu\alpha} \tag{13b}$$

as the basic "particle" equations determined by $T^{\mu\alpha}{}_{*\alpha} = 0$, where the last equality in (13a) follows from the symmetry of $T^{\mu\alpha}$, i.e., $t^{\mu\alpha} = t^{\alpha\mu}$. This is valid for any symmetric $T^{\mu\alpha}$ satisfying $T^{\mu\alpha}{}_{*\alpha} = 0$.

Now specialize to matter and not radiation. Then $T^{\lambda}{}_{\lambda} \neq 0$ so $t^{\lambda}{}_{\lambda} \neq 0$. Hence (13b) gives

$$t^{\lambda} r_{\lambda} \neq 0 \tag{14}$$

Symmetry of $t^{\mu\alpha}$ requires

$$t^{\mu} r^{\alpha} = t^{\alpha} r^{\mu} \tag{15}$$

$$t^{\mu} r^{\lambda} r_{\lambda} = t^{\lambda} r_{\lambda} r^{\mu} \tag{16}$$

Since $t^{\mu} \neq 0$ by construction ($T^{\mu\alpha} \neq 0$ on C) and since $r^{\mu} \neq 0$ (every world line has a tangent) then (14) and (16) require

$$r^{\lambda} r_{\lambda} \neq 0 \tag{17}$$

i.e., C is never null. Hence C is either always timelike or always spacelike. By definition *particles* travel along *timelike* curves so C is timelike. Since $r^{\lambda} r_{\lambda}$ and $t^{\lambda} r_{\lambda}$ are never zero one can define a new strictly monotone parameter σ along C by

$$\frac{d\sigma}{d\kappa} \equiv \frac{r^{\lambda} r_{\lambda}}{r^{\alpha} t_{\alpha}} \tag{18}$$

giving

$$t^{\mu} = \frac{d\xi^{\mu}}{d\sigma}, \qquad t^{\mu}{}_{*\alpha} t^{\alpha} = 0 \tag{19}$$

from (16), (5), and (13a).

Finally, one must determine the *meaning* of t^{μ}. From (12) and (6)

$$t^{0} = \frac{t^{00}}{r^{0}} = \int_{V} T^{00} \, dV = N p^{0} \tag{20}$$

where N is the number of particles inside V with p^0 the energy per particle. Notice that our assumption that $T^{\mu\alpha}$ is nonzero only inside a very thin world-tube means that the particles inside V have almost the same four-momentum p^μ. Hence, define a new path parameter λ such that

$$p^\mu \equiv \frac{dx^\mu}{d\lambda} = \frac{d\sigma}{d\lambda} t^\mu \equiv N^{-1} t^\mu \tag{21}$$

By construction one has

$$\omega^\alpha p_\alpha = E = m\gamma, \qquad \gamma^{-2} \equiv 1 - v^2 \tag{22}$$

if ω^α is an observer four-velocity and v is the relative velocity between particle and observer. Substitution of (21) into (19) and noting that $\Pi(N) = 0$ gives

$$p^\mu{}_{*\alpha} p^\alpha = -p^\mu d \ln N / d\lambda = -p^\mu p^\alpha N_{,\alpha}/N \tag{23}$$

Use of $p^\mu = mu^\mu$ in (23) gives

$$u^\mu{}_{*\alpha} u^\alpha + (\ln m + \ln N)_{*\alpha} u^\alpha u^\mu = 0 \tag{24}$$

and contraction with u_μ shows that

$$(\ln m + \ln N)_{*\alpha} u^\alpha = 0 \tag{25}$$

so one obtains the in-geodesic equation (Adams, 1982; Dirac, 1973)

$$u^\mu{}_{*\alpha} u^\alpha = 0 \tag{26}$$

as the equation of motion of free particles. Notice that this applies both to nonreplicating classical particles and to (possibly) replicating quantum particles. This is equivalent to the statement that classical particles do not replicate; their masses increase in A-units (Adams, 1982). The masses of quantum particles do not increase in A-units; they replicate (Adams, 1982).

2.2. Photon Propagation Equation. Having demonstrated the field-to-particle technique by obtaining the in-geodesic equation for matter, I now apply the same technique to photons. Again one starts with a symmetric energy tensor satisfying $T^{\mu\alpha}{}_{*\alpha} = 0$. The only difference is that now $T^\lambda{}_\lambda = 0$. Hence all the above results are valid down through (13).

Since $T^\lambda{}_\lambda = 0$ then $t^\lambda{}_\lambda = 0$ so from (13b)

$$t^\lambda r_\lambda = 0 \tag{27}$$

and (16) gives

$$r^{\lambda}r_{\lambda}=0 \tag{28}$$

i.e., C is null. Photons must travel on null curves. Contracting (15) with t_{μ} and using (27) shows that t^{μ} is null. Hence (27) and (28) require that r^{μ} and t^{μ} be proportional,

$$t^{\mu}=Ar^{\mu} \tag{29}$$

for some scalar function $A(x)$. Since t^{μ} and r^{μ} are never zero $A(x)\neq 0$. Define a new strictly monotone parameter σ on C such that

$$\frac{d\sigma}{d\kappa}\equiv A^{-1}(x) \tag{30}$$

$$t^{\mu}=\frac{d\kappa}{d\sigma}r^{\mu}=\frac{d\xi^{\mu}}{d\sigma} \tag{31}$$

Then (13a) becomes

$$t^{\mu}{}_{*\alpha}t^{\alpha}=0 \tag{32}$$

which shows that C is a null in-geodesic.

Again, one must determine the *meaning* of t^{μ}. Again one finds (20) and hence (21) except that now $N=N_{\gamma}$ is the number of photons inside V and p^{0} is the energy per photon. By construction one has

$$\omega^{\alpha}p_{\alpha}=E_{\gamma}=h\nu \tag{33}$$

if ω^{α} is an observer four-velocity, h is Planck's constant (Adams, 1982),

$$h=h_{A}(\beta/\varphi)^{g-2} \tag{34}$$

and ν is the observed photon frequency. Again one finds (23) for photons. However, since p^{μ} is null the steps leading to (26) fail. Equation (23) is the form of the propagation equation for photons in terms of the *a priori* unknown photon replication rate. Writing

$$N_{\gamma}=N_{\gamma 0}(\varphi/\varphi_{0})^{a} \tag{35}$$

where a is constant gives

$$p^{\mu}{}_{*\alpha}p^{\alpha}=-ap^{\mu}p^{\alpha}\varphi_{*\alpha}/\varphi \tag{36}$$

where $\varphi_{,\alpha} = \varphi_{*\alpha}$ by (1b). Equations (33) and (36) together form the photon propagation equation which was the main objective of this section.

The derivation of (36) illustrates a vital point which is usually overlooked in the literature. One often reads statements to the effect that since null geodesics are invariant under scale transformations, the photon propagation equation can always be written in the form (32), where t^μ is the photon four-momentum, σ is the path parameter, and

$$t^\mu = \frac{dx^\mu}{d\sigma} \tag{37}$$

From the above derivation it is clear that this statement is false since (33) was neglected. The requirement that (33) hold fixes the photon path parameter to within an additive constant. Thus, while (36) can always be transformed into (32) by a rescaling of the path parameter, the physical content of (36) and (32) is completely different. In (36) p^μ is the photon four-momentum. In (32) t^μ is not the photon four-momentum.

3. RED-SHIFT LAW

Having obtained the photon propagation equation one can now derive the cosmological red-shift law. Taking the cosmological metric

$$d\tau^2 = dt^2 - R^2(t) h_{ij} dx^i dx^j \tag{38}$$

$$\Gamma^i_{00} = \Gamma^0_{00} = \Gamma^0_{0i} = 0, \qquad \Gamma^i_{0j} = \delta^i_j \dot{R}/R \tag{39}$$

where in (39) $\dot{R}$ denotes the time derivative of $R(t)$, one finds

$$\begin{aligned} -ap_0 p^\alpha (\ln\varphi)_{,\alpha} &= -ap_0 d\ln\varphi/d\lambda \\ &= p_{0,\alpha}p^\alpha - p_\alpha \Gamma^\alpha_{0\rho} p^\rho + (2-g) p_0 (\ln\beta)_{,\alpha} p^\alpha \\ &= dp_0/d\lambda + p_0 p^0 \dot{R}/R + (2-g) p_0 d\ln\beta/d\lambda \\ &= dp_0/d\lambda + p_0 d\ln R/d\lambda + (2-g) p_0 d\ln\beta/d\lambda \end{aligned} \tag{40}$$

$$p_0 R(\beta/\varphi)^{2-g} \varphi^{2+a-g} = \text{const} \tag{41}$$

Relative to a comoving observer ($\omega^\alpha = \delta^\alpha_0$) one has

$$p_0 = h\nu = h_A \nu (\beta/\varphi)^{g-2} \tag{42}$$

from (33) and (34) so

$$\nu R\varphi^{2+a-g}=\text{const} \tag{43}$$

is the required cosmic red-shift law. This shows conclusively that it is not possible to specify independently the photon replication law (35) and the red-shift law (43). Unless the photon replication law is specified, the photon propagation equation and hence the red-shift law are completely unknown.

It is of interest to examine the flat space-time limit of (38) in the LNH approximation. Then the red-shift law becomes

$$\nu=\nu_0(t/t_0)^{1-(2+a)/g} \tag{44}$$

which says that if $2+a\neq g$ the color of a laser beam changes as it propagates! I will return to this point later.

4. BLACK-BODY RADIATION. FREE-PHOTON DISTRIBUTION FUNCTION[3]

The distribution function $f(x, p)$ for particles or photons is the number density of particles or photons in phase space. This can be defined as

$$f\equiv N/VP \tag{45}$$

where an observer sees N particles or photons occupying the physical volume V and having local Lorentz momentum components $\mathbf{p}$ in the range

$$\mathbf{p}_0-\frac{\Delta\mathbf{p}}{2}<\mathbf{p}<\mathbf{p}_0+\frac{\Delta\mathbf{p}}{2} \tag{46}$$

Then $P\equiv\Delta p^x\Delta p^y\Delta p^z$. Since $\Pi(V)=3$ from (1c)

$$\Pi(P)=\tfrac{3}{2}\Pi(g_{\mu\nu}p^\mu p^\nu)=3\Pi(M)=3(1-g) \tag{47}$$

one finds

$$\Pi(f)=3g-6 \tag{48}$$

For an ensemble of free particles or photons (no collisions) one seeks to

[3]Misner et al. (1973).

determine

$$\frac{df}{d\lambda} = f\frac{d\ln N}{d\lambda} - f\frac{d\ln VP}{d\lambda} \tag{49}$$

along the path of the ensemble of free particles or photons.

The free-photon propagation law is taken as (36) giving

$$\frac{dp^{\hat{\mu}}}{d\lambda} = p^{\hat{\mu}}\frac{d\ln(\beta/\varphi)^{g-2}}{d\lambda} - p^{\hat{\mu}}\frac{d\ln\varphi^{a+2-g}}{d\lambda} \tag{50}$$

in a local Lorentz frame where λ satisfies (33) and (21). Examine a very small bundle of photons occupying a rectangular phase space region at λ. Since the phase space region is small the photons are all near each other in coordinate space and in momentum space. Hence one is considering a thin beam of almost monochromatic photons (a laser beam).

A typical photon which was at $(\mathbf{x},\mathbf{p})$ at λ has moved to the point

$$(\mathbf{x}+\mathbf{p}d\lambda, \mathbf{p}+A\mathbf{p}d\lambda) \tag{51}$$

$$A = (g-2)\frac{d\ln(\beta/\varphi)}{d\lambda} - (a+2-g)\frac{d\ln\varphi}{d\lambda} \tag{52}$$

at $\lambda + d\lambda$ from (50). The phase space volume at λ is

$$VP|_{\lambda} = \Delta x \Delta y \Delta z \Delta p^x \Delta p^y \Delta p^z \tag{53}$$

while from (51) the phase space volume at $\lambda + d\lambda$ is

$$VP|_{\lambda+d\lambda} = \Delta x \Delta y \Delta z \Delta p^x \Delta p^y \Delta p^z (1+3A d\lambda) \tag{54}$$

giving

$$\frac{dVP}{d\lambda} = 3A = (3g-6)\frac{d\ln(\beta/\varphi)}{d\lambda} - (3a+6-3g)\frac{d\ln\varphi}{d\lambda} \tag{55}$$

so use of (35) and (55) in (49) gives

$$\frac{df}{d\lambda} = f(6-3g)\frac{d\ln(\beta/\varphi)}{d\lambda} + f(4a+6-3g)\frac{d\ln\varphi}{d\lambda} \tag{56}$$

Since f is a coordinate scalar the result (56) is independent of the local

Lorentz frame used to derive it. Further, from (48) one can write (56) as

$$f_{*\alpha}p^{\alpha}+\frac{\partial f}{\partial p^{\alpha}}\frac{dp^{\alpha}}{d\lambda}=4afp^{\alpha}\varphi_{*\alpha}/\varphi \tag{57}$$

which shows that (56) is units covariant as required.

Two points are immediately evident from (56). First, the distribution function depends on β. This is nothing more than a restatement of the fact that f is not a pure number.

The second point is that f need not preserve its form as the photons move in the absence of collisions. In standard physics the right side of (56) is zero (this follows with $\beta=\varphi=1$). Hence, in standard physics in the absence of collisions if $f=f_0$ at $\lambda=\lambda_0$ then $f=f_0$ for all λ. For example, if f is a Planck distribution at $\lambda=\lambda_0$

$$f_0=B\left(e^{h_0\nu_0/k_0T_0}-1\right)^{-1},\qquad B=\text{const} \tag{58}$$

then for any subsequent λ

$$f=B\left(e^{h\nu/kT}-1\right)^{-1},\qquad \frac{h\nu}{kT}\equiv\frac{h_0\nu_0}{k_0T_0} \tag{59}$$

However, in general (56) shows that if $f=f_0$ at $\lambda=\lambda_0$ then

$$f=B(\varphi/\varphi_0)^{4a+6-3g}(\beta\varphi_0/\beta_0\varphi)^{6-3g}\left(e^{h\nu/kT}-1\right)^{-1},\qquad \frac{h\nu}{kT}\equiv\frac{h_0\nu_0}{k_0T_0} \tag{60}$$

for any subsequent λ.

The 2.7 K cosmic photon distribution is measured in A-units. If this distribution is in fact cosmological in origin, and if it in fact has been propagating freely without collisions for most of the age of the Universe, then from its observed Planck distribution today one concludes that in A-units the form of the distribution function must be preserved. Setting $\beta=\varphi$ (A-units) in (60) shows that this condition uniquely determines the photon replication rate to be

$$a=\tfrac{3}{4}(g-2) \tag{61}$$

The significance of (61) should not be underestimated. It was obtained directly by correlating observation with theory with no input from LNH except for the units covariant formalism. Further, I have found no other way to obtain (61). It seems that if the 2.7 K cosmic radiation did not exist this formulation of LNH would have had to stop at this point.

Finally, I derive the evolutionary behavior of an initially black-body photon energy density in an isotropic, homogeneous Universe. Consider the number distribution function for photons per unit volume per unit frequency interval per unit solid angle $n(\nu, t)$. Now

$$\begin{aligned} dN &= f\,dV\,dP = fh^3\nu^2 dV\,d\nu\,d\Omega \\ &\equiv n(\nu, t)\,dV\,d\nu\,d\Omega \end{aligned} \tag{62}$$

$$\begin{aligned} n(\nu, t) &= h^3\nu^2 f \\ &= h_A^3\nu^2 f_0(\varphi/\varphi_0)^{4a+6-3g} \end{aligned} \tag{63}$$

from $\omega^\mu p_\mu = h\nu$, (34), (56), and renormalizing h so that $h_0 = h_A$. From (63) the energy density per unit frequency interval for an isotropic photon distribution is

$$\begin{aligned} \rho_\gamma(\nu, t) &= 4\pi h\nu n(\nu, t) \\ &= 4\pi h_A^4 f_0 \nu^3(\beta\varphi_0/\varphi\beta_0)^{g-2}(\varphi/\varphi_0)^{4a+6-3g} \end{aligned} \tag{64}$$

where again (34) was used. With f_0 given by (58) integration of (64) over ν gives the total radiation density in an isotropic photon black-body distribution as

$$\begin{aligned} \rho_\gamma &= \int_0^\infty \rho_\gamma(\nu, t)\,d\nu \\ &= \rho_{\gamma 0}(\varphi/\varphi_0)^{4a+8-4g}(\beta_0/\beta)^{2-g}(\nu/\nu_0)^4 \end{aligned} \tag{65}$$

where (59) was used. Use of the red-shift law (43) gives

$$\rho_\gamma = \rho_{\gamma 0}(R_0/R)^4(\beta_0/\beta)^{2-g} \tag{66}$$

as required.

Notice that (66) is independent of the photon replication rate. This is to be expected since ρ_γ is the classical radiation density. Since classically photons do not exist, only Maxwell's equations, the fact that (66) says nothing whatsoever about photons is a self-consistency check of the formalism.

5. PREDICTION OF THE N_γ/N_m RATIO

One of the remarkable results of this paper is the fact that the existence of the 2.7 K cosmic radiation can be used to fix the photon replication rate as (61). An even more remarkable result is the prediction of the N_γ/N_m ratio. From Paper I

$$N_m = N_{m0}(\varphi/\varphi_0)^{g-1} \tag{67}$$

so (35) gives

$$N_\gamma/N_m = (N_{\gamma 0}/N_{m0})(\varphi/\varphi_0)^{a+1-g} \tag{68}$$

Use of (61) gives

$$N_\gamma/N_m = (N_{\gamma 0}/N_{m0})(\varphi/\varphi_0)^{-(2+g)/4} \tag{69}$$

$$\simeq (N_{\gamma 0}/N_{m0})(t/t_0)^{-(1+2/g)/4} \tag{70}$$

where (70) follows from the LNH approximation for φ (Adams 1982).

For additive or local creation $g \simeq +1$ while for multiplicative creation $g = -1$. (Adams, 1982) From (70)

$$N_\gamma/N_m \simeq t^{-3/4} \qquad (g = +1) \tag{71a}$$

$$N_\gamma/N_m \simeq t^{1/4} \qquad (g = -1) \tag{71b}$$

But (71b) is precisely one of the conclusions of LNH (Adams, 1982). Hence the assumption of multiplicative creation allows one to develop a formalism consistent with observation which actually *predicts* one of the large numbers. Notice that the condition (71b) was nowhere imposed prior to its being derived here. This should be contrasted with (67), which, while it can be derived from this theory, was actually used in Paper I to set up the formalism in the first place so the argument is circular.

This result more than any other leads one to believe that both (61) and (71b) have something to do with the way Nature works, if this formalism is applicable at all. In future papers the parameter g will be carried along in the formalism since it keeps track of mass units. However, from now on the canonical LNH will have

$$\text{MULTIPLICATIVE CREATION} \tag{72a}$$

$$g = -1 \tag{72b}$$

$$N_m = N_{m0}(\varphi_0/\varphi)^2 \tag{72c}$$

$$N_\gamma = N_{\gamma 0}(\varphi_0/\varphi)^{9/4} \tag{72d}$$

$$\varphi/\varphi_0 \simeq t_0/t \tag{72e}$$

$$G = G_0(\varphi/\varphi_0) \tag{72f}$$

$$\frac{\nu}{\nu_0} = \frac{R_0}{R}\left(\frac{\varphi_0}{\varphi}\right)^{3/4} \tag{72g}$$

and the free-photon distribution function is preserved in A-units.

6. DISCUSSION

This paper has presented details concerning properties of electromagnetic radiation based on the units covariant formalism as a means to incorporate LNH into physics. It was shown that the photon propagation equation depends on the photon replication rate. It was also shown that this rate can be determined in terms of the parameters of the theory if one imposes the requirement that the form of the free-photon distribution function be preserved in A-units, as in standard physics. This requirement seems to be dictated by the observation that the 2.7 K radiation is cosmological and has a black-body distribution. Even more remarkable is the fact that for a unique choice of the parameter g, viz., $g = -1$ corresponding to multiplicative creation, this theory predicts the same time dependence for N_γ/N_m as deduced from LNH. This particular large number was never used in the theory prior to this development. This lends strong support to the assertion that if this units covariant formalism has any validity at all, the case of multiplicative creation ($g = -1$) seems to be preferred by Nature.

Knowledge of the photon propagation equation allows the red-shift law to be derived. Use of the photon creation rate (61) allows the red-shift law (43) to be written as

$$\nu/\nu_0 = (R_0/R)(\varphi_0/\varphi)^{(2-g)/4} \simeq (R_0/R)(t/t_0)^{3/4} \tag{73}$$

where the last equality is the prediction of the canonical LNH. This appears to contradict "common sense" since everyone knows that

$$\nu R = \text{const} \tag{74}$$

can easily be derived by counting wave crests (Schrödinger, 1957; Misner et al., 1973, p. 777). However, (74) is valid for a classical wave, not necessarily for photons. If photons in fact behave like classical waves then (74) is correct. Some workers in this subject believe that photons *must* behave like classical waves. This condition is then used in (73) to fix g by requiring $g = +2$ (Canuto and Hsieh, 1978).

However, there is nothing in physics which *requires* photons to behave like classical waves. Maxwell's equations certainly do not impose such a requirement since they do not contain photons at all. There is no experimental evidence on the subject other than that photons behave like classical waves over path lengths of a few astronomical units to within the accuracy of measurement. What one is essentially doing here is asking how the color of a laser beam changes over cosmological distances relative to comoving observers. This is completely different from asking how the frequency of classical waves changes over cosmological distances.

In standard physics one writes

$$p_\mu = hk_\mu = h\theta_{,\mu} \tag{75}$$

where θ is the phase of the wave and p_μ is the photon momentum. Here is where the question of normalization of path parameter enters. As discussed in Section 2, the path parameter is specified up to an additive constant by

$$\omega^\alpha p_\alpha = h\nu \tag{76}$$

for an observer with four-velocity ω^α. There is no guarantee that the four-momentum of (76) is related to the gradient of the phase by (75). In this formalism there is a rescaling of the path parameter which involves φ.

In standard physics one has no reason to treat photons as anything other than classical waves. In this units covariant formalism with LNH it is mandatory that photons be treated as photons, not necessarily as classical waves. Further, one has the φ "field" which in principle is capable of affecting photon color. This is the first example of how φ enters the formalism nontrivially so as to affect quantum dynamics.

In all prior applications φ entered by setting $\beta = \varphi$ in the units covariant formalism. Here φ enters directly. The fact that this is the first application directly involving quantum phenomena (photons) is not accidental. φ is inherently connected with quantum physics, not with classical physics. As the theory is pushed closer to the quantum regime φ will enter directly more often.

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